

THESIS

AQUIFER TEST METHODS TO ESTIMATE TRANSMISSIVITY AND WELL LOSS VIA A
SINGLE PUMPING WELL

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ABSTRACT

AQUIFER TEST METHODS TO ESTIMATE TRANSMISSIVITY AND WELL LOSS VIA A SINGLE PUMPING WELL

Reducing the energy, environmental impacts, and costs required to produce groundwater from wells is emerging as a critical concern in the modern world. Establishing and maintaining pumping wells with minimal excess drawdown is thus important. Numerous methods have been devised to quantify the aquifer and well contributions of the total drawdown in a pumping well. The most common single-well methods involve step-drawdown test analyses that are inherently subjective and cumbersome. The limitations are overcome here using simple, analytical methods following the equations of Theis, Jacob, and Rorabaugh to estimate an aquifer's transmissivity and a well's well-loss parameters and excess drawdown due to well-loss effects. The major steps involve derivative analysis, solving systems of equations, and making elementary assumptions. The proposed methods analyze data from independent constant-rate tests at single pumping wells. In fact, Jacob's well-loss coefficient of $4.6 \times 10^{-7} \frac{\text{day}^2}{\text{m}^5}$ and $0.39 \times 10^{-7} \frac{\text{day}^2}{\text{m}^5}$ were estimated at a pumping well via Jacob's (1947) traditional step test analysis and the proposed method, respectively. The reduced subjectivity of the proposed method suggests it produced the more accurate estimate. The required energy and associated economic and environmental equivalences of pumping the well loss are then calculated to suggest when further development of a new well or the rehabilitation of a preexisting well is needed. The overall goal of this thesis is to advance the proposed methods for more straightforward and objective analyses of aquifer test data in academia and industry to promote the energy efficiency of groundwater production.

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1. INTRODUCTION

1.1 Overview

Groundwater production for water supply, irrigation, and industry has been increasing with a growing world population (Konikow 2015). Emerging concerns of expanding groundwater production include the energy required to lift water out of aquifers and the associated economic and environmental costs. For example, as much as one third of the total electrical power use and carbon footprint for a house between Denver and Colorado Springs can be attributed to the energy needed to lift water from the underlying Denver Basin Aquifers (Ronayne et al. 2017).

The principle factors that govern the energy needed to produce groundwater are total dynamic head (TDH) and the efficiency of the mechanical equipment including pumps, motors, and motor drives. A well system's TDH [L] is the total head required to lift water out of the aquifer to a point of use while meeting pressure needs and overcoming all head losses throughout the system. As an example, the head components of a domestic groundwater well are illustrated in Figure 1.

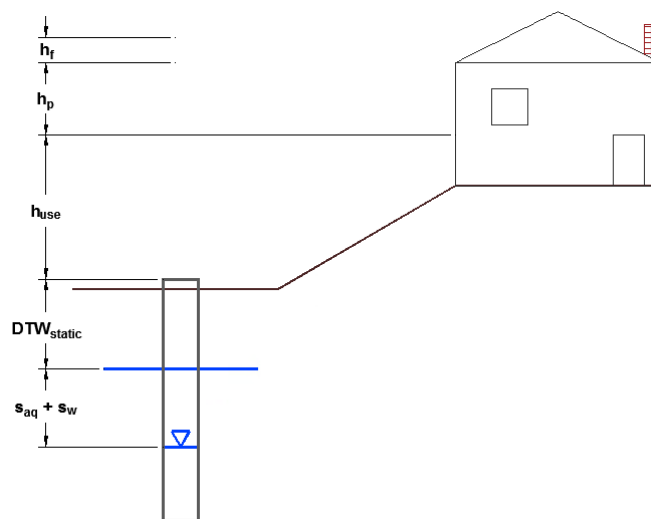


Figure 1 – An example of the hydraulic heads that contribute to total dynamic head (TDH) of a domestic groundwater well system

Correspondingly, TDH can be defined as:

$$TDH = h_f + h_p + h_{use} + DTW_{static} + s_{aq} + s_w \quad (1)$$

where h_f [L] is the friction losses contributable to conveyance, h_p [L] is the pressure head required for sufficient discharge at the point of use (i.e., the house's top-floor faucet), h_{use} [L] is the elevation change from the well head to the point of use, DTW_{static} [L] is the depth from the well head to the static groundwater level prior to pumping, s_{aq} [L] is the drawdown associated with pumping water from the aquifer, and s_w [L] is the excess drawdown (or "well loss") associated with nonlaminar flows through the well during pumping.

This thesis has two goals. The first is to develop straightforward methods to accurately estimate the aquifer's transmissivity about a single pumping well. The second is to advance simple and less subjective methods for quantifying a pumping well's s_w to support efforts that reduce energy usage and the carbon footprint associated with groundwater production. Specifically, minimizing s_w contributions to the total drawdown (s_{total} [L]) within a pumping well will reduce the distance between the pumping water-level and the point of use. Figure 2 depicts a conceptual model of the distribution of drawdowns about a pumping well.

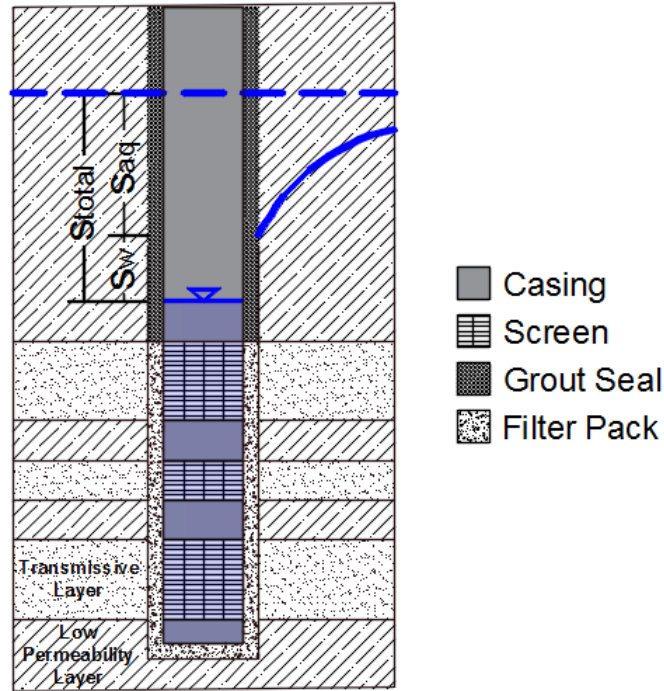


Figure 2 – Conceptual model of aquifer and well losses about a pumping well

Two methods are presented using the observed rate of change in s_{total} during pumping, called derivative drawdown data, to estimate the aquifer's transmissivity (T [L^2/T]), which is the primary aquifer property needed to estimate s_{aq} during long-term pumping. The first approach employs derivative analysis via a central finite difference scheme. The other takes the temporal derivative of a simple logarithmic trendline equation of the drawdown data. Both are used to analyze an aquifer test from a confined aquifer but should also be applicable for an unconfined aquifer once delayed drainage has passed and if the change in the saturated thickness remains small during pumping (Theis 1935). Theis' curve matching and Cooper-Jacob's straight-line approximation can obtain T estimates, but these methods are subjective. For example, several methods were applied to Clark's (1977) step-drawdown test data and returned T values in the range of 250-380 m^2/day (Kawecki 1995).

Next, two methodologies are advanced to quantify s_w using drawdown data from a single pumping well. The first methodology solves for Jacob's well-loss coefficient (C_J [T^2/L^5]) while the other resolves Rorabaugh's well-loss coefficient (C_R [T^n/L^{3n-1}]) then Rorabaugh's well-loss exponent (n [-]). Both methodologies utilize independent constant-rate aquifer tests. Finally, s_w is resolved analytically and, thus, the proposed methods bypass the subjective nature of standard step-drawdown test analyses initially advanced by Jacob (1947). The methods here are also mathematically straightforward, so the complexity that is apparent with Rorabaugh's (1953) trial-and-error straight line approach and Sheahan's (1971) family of type curves are overcome. Moreover, the proposed methods can be considered economical as they do not require analyzing data from observation wells, which can be expensive to design, install, and develop.

The proposed methods effectively bypass the subjective, complex, and cumbersome signatures of standard single-well aquifer test analyses by building on the principal drawdown equations of Theis (1935), Jacob (1946), and Rorabaugh (1953). The steps involved should be straightforward enough for academia and industry to easily adopt the proposed methods in order to obtain more accurate aquifer and well parameter estimates about a single pumping well. Economic and environmental analyses are also conducted to ensure efficient well performance. Ultimately, absolute s_w values will prove useful for characterizing the efficacy of new groundwater well installations and rehabilitation efforts of preexisting wells. For example, development of a new well may be deemed sufficient if s_w and the associated economic and environmental impacts of pumping are low. Similarly, rehabilitation of a preexisting well will be suggested once the s_w and associated economic and environmental effects become too large.

1.2 Objectives

The following work aims to develop straightforward methods to accurately estimate the aquifer's transmissivity, Jacob's well-loss coefficient or Rorabaugh's well-loss coefficient and well-loss exponent, and subsequent excess drawdown (or "well loss") due to nonlaminar flows at the design pumping rate. The proposed methods highlight the efficacy of using well loss as a well performance metric. Ultimately, the methods can quantify and ensure well loss in pumping wells are minimized to reduce the energy and associated economic and environmental costs of groundwater production.

1.3 Organization

The second section of this thesis introduces the principle drawdown equations including those of Theis, Jacob, and Rorabaugh. Current method's limitations and other background work are reviewed to support the thesis' subsequent efforts. The third section derives the aquifer and well parameter solutions. The fourth section applies the proposed solutions to both field and synthetic data to serve as a useful set of examples. The author also includes his interpretation and recommendations to utilize the solutions effectively. The fifth section summarizes this thesis' key results and implications while suggesting potential avenues for future efforts.

2. WORK BY OTHERS

This section presents the relevant knowledge of groundwater well hydraulics from the work of others. Definitions and principal equations are established, factors that cause well loss are reviewed, and the limitations of the existing aquifer test methods that estimate aquifer and well parameters are discussed. This review provides a foundation for the subsequent Methods Section.

2.1 Definitions and Principal Equations

Equation 1 and Figure 2 illustrated that the s_{total} at a pumping well can be simply defined as:

$$s_{total} = s_{aq} + s_w \quad (2)$$

Equation 2 is analogous to Jacob's (1947) equation and is regarded as an integration of the hydraulic losses throughout the flow field:

$$s_{total}(Q, t) = B(t)Q + C_J Q^2 \quad (3)$$

where $B(t)$ is the aquifer-loss coefficient [T/L^2] and Q is the well's pumping rate [L^3/T], thus $B(t)Q$ is the "aquifer-loss term" [L] which actually accounts for all laminar flows in the aquifer-well system (K ergaard 1982; Kawecki 1995). Laminar flows typically occur in the formation, skin zone, and gravel pack. Next, C_J is Jacob's well-loss coefficient [T^2/L^5], thus $C_J Q^2$ is Jacob's "well-loss term" [L] that represents the head losses due to turbulent flows in the system. Turbulent flow typically occurs through a well's screens and inside the casing. Jacob assumed the well-loss exponent, n [-], is equal to 2 as the observed head loss for fully-developed turbulent flow in a pipe is a function of Q^2 (Jacob 1947). As a result, Jacob's equation does not account for the non-linear laminar flow regime, which could describe flows through the formation

immediately about a well, the formation damaged by drilling fluids (skin zone), and/or gravel pack (Houben 2015). Nonetheless, Jacob's equation has been found to have a physical background for reasonably homogenous regions about a well (Kærsgaard 1982).

Rorabaugh's (1953) equation for s_{total} , on the other hand, has an additional fitting parameter that is used when Equation 3 is found to be inadequate to model a time series of drawdown data:

$$s_{total}(Q, t) = B(t)Q + C_R Q^n \quad (4)$$

where C_R is Rorabaugh's well-loss coefficient [T^n/L^{3n-1}], so $C_R Q^n$ is Rorabaugh's "well-loss term" [L]. Rorabaugh describes well-loss effects by leaving the well-loss exponent as an unknown parameter to be estimated. The n is reported to range between 2.4 and 2.8 with an average of 2.5 for field data (Rorabaugh 1953). Some have referenced Lennox's 1966 work to infer that n can even be as high as 3.5, but Lennox, two years later, stated that the data were inadequate to put "too great a reliance on these results...some later test data which were unavailable for the original paper have given values of $[n]$ less than 3" (Lennox 1968).

For both Equation 3 and 4, $B(t)Q$ is time-dependent and can be represented by the Theis equation for use in ideal confined aquifer settings or late-time, unconfined aquifer settings if the change in the saturated thickness remains small during pumping (Theis 1935):

$$s_{aq}(Q, t) = B(t)Q = \frac{Q}{4\pi T} \int_{u(t)}^{\infty} \frac{e^{-m}}{m} dm \quad (5)$$

$$u(t) = \frac{r^2 S}{4Tt}$$

where T is the aquifer's transmissivity [L^2/T], S is the aquifer's storage coefficient [-], r is the radial distance from the pumping well to a point of interest [L], t is the elapsed time since

pumping began [T], and m is the variable of integration. Since the proposed methods evaluate the drawdown at the pumping well, the effective well radius (r_e [L]) can be substituted in for r in Equation 5 (Jacob 1947). The pumping well's r_e implicitly incorporates laminar well-loss effects into s_{aq} (Kaecki 1995). Again, s_{aq} actually represents all laminar flows in the aquifer-well system.

Comparing Equation 2 and 3 allows s_w , the excess drawdown during pumping (or “well loss”), to be modeled as:

$$s_w(Q) = C_J Q^2 \quad (6)$$

and, similarly, the likeness between Equations 2 and 4 allows for Rorabaugh's s_w model:

$$s_w(Q) = C_R Q^n \quad (7)$$

An advantage of Jacob's s_w model (Equation 6) is that it only has one unknown parameter that needs to be estimated. The advantage of Equation 7 is that the two unknown parameters can provide the model more degrees of freedom to better fit the observed data.

2.1.1 Derivative Analysis

Following Lewis et al. (2016), s_w effects at a pumping well can be removed via derivative analysis. During a constant-rate test, Q , C_J/C_R , and n are assumed to be constant, thus:

$$\frac{ds_w}{dt} = 0 \quad (8)$$

which makes the temporal derivative of Equation 2 [$s_{total} = s_{aq} + s_w$]:

$$\frac{ds_{total}}{dt} = \frac{ds_{aq}}{dt} \quad (9)$$

Similar to the solution obtained by Straface (2009), the derivative of Theis' equation (Equation 5) with respect to time (via PTC Mathcad 2015) is:

$$\frac{ds_{aq}}{dt} = \frac{Q e^{-\frac{r_e^2 S}{4Tt}}}{4\pi T t} \quad (10)$$

It is important to note that the improper integral of Theis' equation was cleared since the operations of differentiation and integration negate each other. Equation 10 can be simplified to resolve transmissivity estimates from a single pumping well completed in a confined aquifer. Section 3.1.1 develops this method further.

2.2 Well-Loss Contributors

Well-loss effects create nonlaminar flows that, in turn, cause s_w . Well-loss effects are predominately dictated by the well's design, drilling, and development (Clark 1977; Sterrett 2007; Hemenway 2018). Groundwater flow through a well's screen and casing can also increase s_w , especially for deep wells with limited screen open-area (Clark 1977; Kawecki 1995; Sterrett 2007). Furthermore, other effects can cause the well loss to substantially increase over time (Houben et al. 2018). The plethora of well-loss effects warrants a review of each, individual contributor.

To start, the design of a groundwater production well plays a large role in governing s_w during pumping. As one example, open area is important for effective well development (Sterrett 2007). Sufficient open area also ensures groundwater entrance velocities remain below 3 cm/s to maintain laminar flow across the screens (Wendling et al. 1997; Sterrett 2007). If there is not enough open area, then flow can become turbulent and cause well loss.

Secondly, drilling can significantly contribute to well loss in the case of near-well formation damage. The degree of formation damage is highly dependent on the used drilling method. In the case of high-capacity groundwater wells, the most common drilling method is mud rotary where fluid pressure in a well is greater than the adjacent formation pressure (overbalanced drilling). As a result, drilling fluid can be easily lost to the aquifer. The formation near the well is now considered damaged with an area of reduced permeability, otherwise known as a “skin zone”.

Even wells drilled without drilling fluids can develop skin zones (Hanna et al. 2003; Sterrett 2007). Therefore, an objective of development procedures is to remove the skin zone, although it is difficult to remove 100% of the drilling fluids. It is best practice to employ multiple development techniques to enhance development efforts as much as possible (Sterrett 2007). If development is insufficient, well loss will occur through the filter pack and skin zone.

Next, water flowing through a well’s casing is analogous to pipe flow. Thus, the head loss through a well can be modeled with the Darcy-Weisbach equation, which is function of the “pipe length” and square of the flow rate. In-casing well loss, therefore, become significant yet unavoidable in deep wells (Carl 1977; Kawecki 1995) . Even keeping the casing velocity below the recommended 1.5 m/s can still cause moderate friction losses in a well (Sterrett 2007).

Moreover, significant well loss can be contributed to the mechanical compaction of the filter pack and clogging of the filter pack and well screens over time. This compaction and clogging can reduce the filter pack’s porosity and screen open area, which would restrict flow paths and, ultimately, cause well loss during pumping (Houben et al. 2018). Processes that induce clogging can include the deposition of mineral phases (i.e., iron and manganese oxides and calcite), the growth of biofilms, and the invasion of particles (Houben 2003; Houben and Treskatis 2007).

The aquifer's transmissivity can quickly determine the relative importance of well-loss effects in an aquifer-well system. Bierschenk (1963) explains that s_{aq} varies inversely with T . For example, if T is high, s_{aq} is small, therefore s_w can be a relatively large proportion of s_{total} .

Finally, a uniform distribution of inflow to a well seldom occurs in the field (Kærgaard 1982). Given the log-normal distribution of hydraulic conductivity in nature, it follows the inflow could be an order of magnitude greater than the mean. In other words, generally, 90% of the flow can come from 10% of the aquifer section. Heterogeneity of the aquifer or skin layer can further create preferential flow paths, ultimately affecting the vertical inflow distribution across a well's screen (Houben and Hauschild 2011). A similar effect can occur if a well screen has a poor distribution of screen openings (either from improper well design or well screen clogging) that causes excessive convergence flow near the individual openings (Sterrett 2007).

It quickly becomes apparent that there are many possible contributors of well loss in a pumping well. Several of the contributors are interconnected and potentially synergistic, too. As such, the proposed methodologies are important for quantifying s_w at a well's design pumping rate to serve as a well performance metric. When s_w estimates become too large, further development of a new well or rehabilitation of a preexisting well should follow to ensure a community receives and maintains an efficient groundwater well.

2.3 Limitations of Standard Single-Well Methods for Estimating Aquifer and Well Losses

There are several standard methods that strive to estimate the aquifer and well-loss contribution of the total drawdown in a pumping well. Each method has limitations. The most common methods and their limitations are discussed in further detail in this subsection.

The s_{aq} component of s_{total} can be described with the aquifer's properties. Currently, the primary approaches to estimate the aquifer's T about a single pumping well analyze constant-rate test data via Theis curve matching or the Cooper-Jacob straight line approximation (Fetter 2001; Sterrett 2007; Mays 2012). Matching the curves or overlaying the line can be subjective and subsequent results may vary significantly even between seasoned professionals.

The s_w component of s_{total} can be described with the well's parameters. Typically, the Rorabaugh (1953), Hantush-Bierschenk (1964), Sheahan (1971), and Eden-Hazel (1973) techniques are used to resolve a well's C and n (if not assumed to be 2). It is difficult to reliably and consistently obtain each parameter, though, for many reasons.

To start, the four techniques all rely on step-drawdown test data. Step-drawdown test analyses require a minimum of four, but ideally more, pumping steps for reliable estimates of C (Jacob 1947; Rorabaugh 1953; Clark 1977; Singh 2002). Moreover, the measured drawdown after the first step not only represents the pumping effects of itself, but also contains the residual drawdown of the preceding steps (Kærgaard 1982). The data should be corrected for this effect if it is analyzed, but that requires the aquifer's T and S . Also, drawdown estimates beyond the second step may be incorrect due to erroneous extrapolation of drawdown trends (Lennox 1966; Mogg 1969). As a result, step-drawdown test analyses can be highly subjective depending on how a professional likes to procure and analyze step-drawdown test data.

The aforementioned techniques that resolve well parameters can be time-consuming (Jha et al. 2006; Memon et al. 2012). Kruseman and de Ridder (2000) show that the techniques involve many steps, too, which create many opportunities to make mistakes. As a result, computer-based approaches have been developed to analyze aquifer test data more quickly and consistently.

However, these approaches can be quite daunting, especially when the computer code is written in an unfamiliar programming language or uses a complex mathematical operation.

Another limitation of aquifer test methods is the inability to procure S from a pumping well (Kawecki 1995). The presence of well-loss effects along with other processes such as wellbore storage or the expulsion of water from adjacent aquifer layers seem to significantly affect S estimates. T estimates, however, are unbiased by well loss. The effects of well loss on aquifer parameter estimates can be exemplified when the Theis (1935) curve matching and the Cooper-Jacob (1946) straight-line methods analyze synthetic data, depicted in Figure 3 and Figure 4, respectively. Two drawdown series were produced: (1) drawdown due to just aquifer-loss contributions (s_{aq}) and (2) the total drawdown from both aquifer- and well-loss contributions ($s_{aq} + s_w$) in a pumping well.

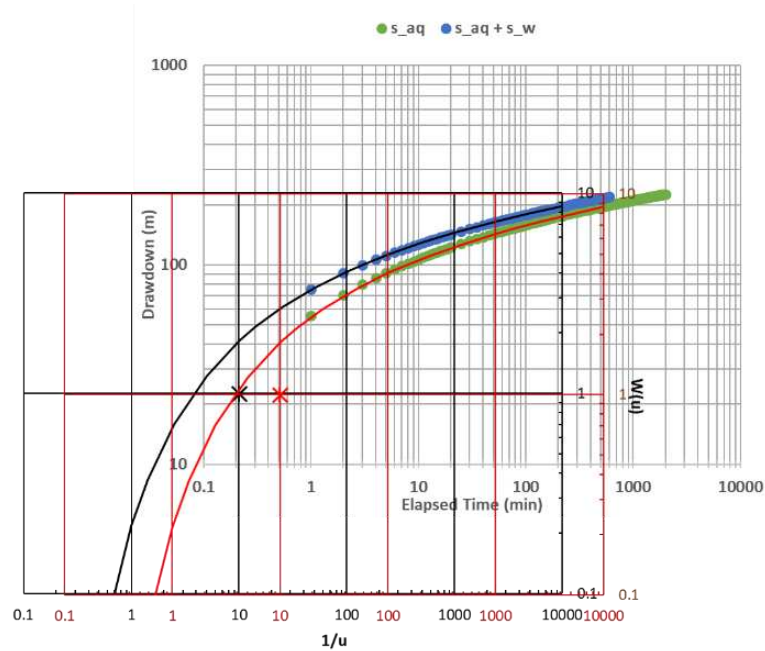


Figure 3 – Well-loss effects on $r_e^2 S$ estimates at a pumping well via Theis (1935) curve matching

Theis' ideal curve in Figure 3 is solely translated left towards a smaller t when well-loss contributions are added to the aquifer drawdown data, so the matched well function ($W(u)$) is the same for each ideal curve. Thus, T is unchanged between the s_{aq} and s_{total} curves following Theis' equation for the aquifer's transmissivity evaluated at a pumping well:

$$T = \frac{Q}{4\pi s} W(u) \quad (11)$$

On the other hand, since the matched t of s_{total} is less than that of s_{aq} , s_{total} 's $r_e^2 S$ estimate would be relatively smaller following Theis' equation for the aquifer's S multiplied by r_e^2 :

$$r_e^2 S = 4Ttu \quad (12)$$

Cooper-Jacob's straight lines in Figure 4 further propound that well loss affects S estimates but not T estimates at a pumping well. Well loss translates the s_{aq} straight line upwards without distorting its slope (Δs over the same log-cycle), so T estimates are unchanged between the s_{aq} and s_{total} lines following Cooper-Jacob's equation for the aquifer's transmissivity:

$$T = \frac{2.3Q}{4\pi \Delta s} \log\left(\frac{t_2}{t_1}\right) \quad (13)$$

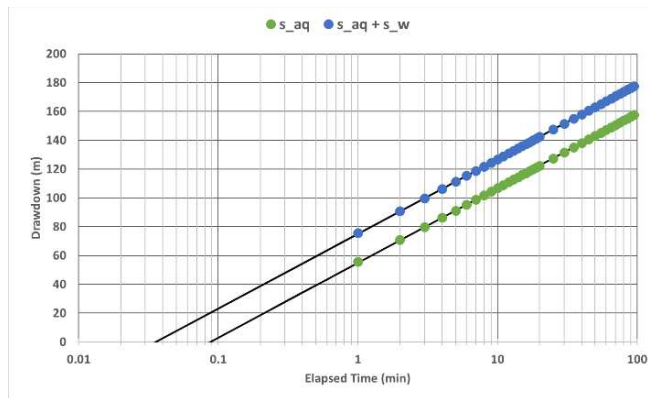


Figure 4 – Well-loss effects on $r_e^2 S$ estimates via the Cooper-Jacob (1946) straight-line method

The estimates of $r_e^2 S$, however, are different between the two lines as the “ t at zero-drawdown”, t_o [L], has changed in Cooper-Jacob’s equation for the storage coefficient multiplied by r_e^2 :

$$r_e^2 S = 2.25 T t_o \quad (14)$$

The $r_e^2 S$ estimate for s_{total} would be smaller than that of s_{aq} since the s_{total} line projects back to a smaller t_o compared to the s_{aq} line. Also, recall the data’s T estimate was unchanged between the two drawdown lines.

Consequently, S cannot be calculated from pumping well drawdown data; perhaps if the total well loss and effective well radius are both known precisely, but this is rarely the case (Clark 1977; Kawecki 1995). Some methods seek to estimate the fusion parameter $r_e^2 S$, but this is not of great use (Lennox 1966; Kawecki 1995). The aquifer’s T and S control groundwater flow, but T is undoubtedly the predominant parameter for well yield at long-term pumping. As a result, S will not be calculated here. The proposed methods are not dependent on S values anyway.

Another major limitation of standard analyses is the use of well efficiency as a well performance metric. Kawecki (1995) summarized the industry’s definition and caveats of well efficiency. Its general form appears to be:

$$E_w = \frac{\text{aquifer loss}}{\text{aquifer loss} + \text{well loss}} * 100\% \quad (15)$$

The aquifer-loss and well-loss terms can be defined as they were for the total drawdown equation to result in:

$$E_w(Q, t) = \frac{s_{aq}(Q, t)}{s_{aq}(Q, t) + s_w(Q)} * 100\% \quad (16)$$

Several authors have pointed out this metric is over relied upon and meaningless as an absolute value (Clark 1977; Kawecki 1995; Shekhar 2006). Well efficiency is more useful for comparing wells of similar design within similar aquifers or assessing a single well through time. Moreover, the elapsed pumping time at which well efficiency is evaluated needs to be quoted because s_{aq} is time-dependent (Lennox 1966; Mogg 1969). There are no current standards to establish such baselines, so well efficiency is not yet a useful indicator of well performance.

Generally, current aquifer test methods are susceptible to subjectivity, complexity, and high costs. Aquifer and well parameters about a pumping well need to be more simply and reliably procured. Therefore, a novel set of straightforward aquifer test methods are developed in the next section.

3. METHODS

This section builds on the foundation laid by the previous Work by Others Section to develop new methodologies to analyze aquifer test datasets. The proposed methods allow one to estimate the aquifer's transmissivity, Jacob's well-loss coefficient, Rorabaugh's well-loss coefficient and well-loss exponent, and the economic and environmental equivalences of pumping well loss.

3.1 Estimating the Aquifer's Transmissivity

3.1.1 Via Derivative Analysis

The aquifer's transmissivity is the predominant parameter for anticipating the performance of production wells. The aquifer's storage coefficient is also important but hardly affects drawdowns during long-term pumping. It is therefore imperative to resolve a reliable T that is representative of the aquifer system and, also, not biased by s_w . Following Lewis et al. (2016) and Equation 10, s_w at a pumping well can be removed via derivative analysis.

In fact, Equation 10 $[\frac{ds_{aq}}{dt} = \frac{Qe^{-\frac{r_e^2 S}{4Tt}}}{4\pi Tt}]$ can be simplified to resolve transmissivity estimates from a single pumping well. Since r_e should be close to the radius of a pumping well, the $\frac{r_e^2 S}{4Tt}$ argument quickly becomes very small (i.e., less than 0.01). Then, $e^{-\frac{r_e^2 S}{4Tt}}$ will be very close to unity. An example of the exponential calculations vs time at a pumping well with aquifer parameters representative of a confined aquifer is shown in Figure 5.

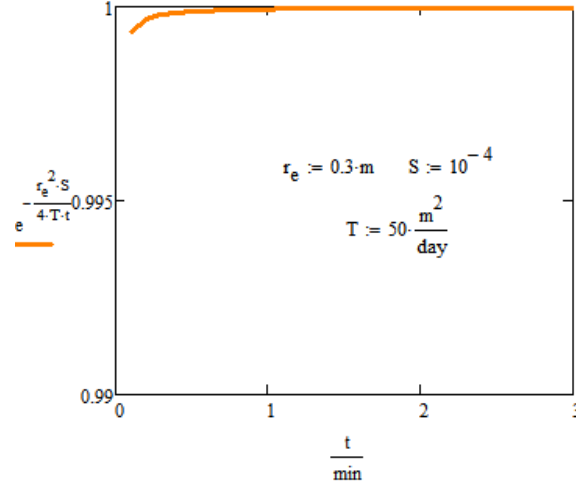


Figure 5 – Exponential calculations of a pumping well with aquifer parameters representative of a confined aquifer

Assuming $e^{-\frac{r_e^2 S}{4Tt}} \cong 1$ for all t leads to a significant simplification (along with Equation 9):

$$\frac{ds_{total}}{dt} \cong \frac{Q}{4\pi T t} \quad (17)$$

Notice the derivative drawdown in a pumping well is not biased by well-loss effects as Equation 17 is not a function of s_w or r_e . For instance, s_w can represent non-laminar well-loss effects such as turbulent flow through the well screen and casing, or non-linear laminar flow through a partially clogged skin zone, filter pack or well screen. Flows through the skin zone or filter pack, however, may be laminar. In that case, r_e embodies those flows as laminar well-loss effects.

Rearranging Equation 17, while substituting in the more aesthetically pleasing variable for the derivative drawdown $[\dot{s}(Q, t) = \frac{ds_{total}}{dt}]$, produces the final solution for the aquifer's transmissivity about a single pumping well:

$$T(t) \cong \frac{Q}{4\pi \dot{s}(Q, t) t} = \frac{Q}{4\pi \Omega} \quad (18)$$

Generally, a confined aquifer's T should be constant through time about a given well location, so the product of the two transient parameters in Equation 18 should result in a constant value for a given Q and T :

$$\dot{s}(Q, t) t = \Omega = \text{constant} \quad (19)$$

Therefore, Equation 18 can be plotted vs t to arrive at the converging value. Otherwise, the average value and a confidence interval of the $T(t)$ estimates should be calculated. For example, the application of Equation 18 on field data produced oscillating $T(t)$ estimates about the average value; Section 4.2.1 provides more details.

Notice the Q does not have to be the same between two different constant-rate tests to resolve the same T via Equation 18. When Q increases, the Ω [L] should correspondingly increase to produce a consistent, final T estimate at a well. More precisely, a larger pumping rate will impart a greater stress on the aquifer, so a well's derivative drawdown will proportionally increase to resolve the same T at a pumping well. Also notice estimates of the aquifer's transmissivity will be independent of nonlaminar well-loss effects as the s_w term is not present in Equation 18.

3.1.2 Via Fitting a Trendline

Another quick, but non-analytical technique to procure T can be done with a trendline. A time series of drawdown data can be smoothed and fit with a natural logarithmic equation via the scatter plot's trendline function. Similarities between a logarithmic best-fit equation and Cooper-Jacob's (1946) approximation of s_{aq} allow the following steps:

$$s_{total}(Q, t) = \frac{Q}{4\pi T} \ln \left(\frac{2.25Tt}{r_e^2 S} \right) + s_w(Q) \quad (20a)$$

$$s_{total}(Q, t) = \frac{Q}{4\pi T} \left[\ln(t) + \ln\left(\frac{2.25T}{r_e^2 S}\right) \right] + s_w(Q) \quad (20b)$$

$$s_{total}(Q, t) = \frac{Q}{4\pi T} \ln(t) + \left(\frac{Q}{4\pi T} \ln\left(\frac{2.25T}{r_e^2 S}\right) + s_w(Q) \right) \quad (20c)$$

$$s_{total}(Q, t) = \tau_1 \ln(t) + (\tau_2) \quad (20d)$$

Equation 20d is a generalized form of Equation 20c and a logarithmic trendline equation using condensed “best-fit equation parameters”, τ_1 and τ_2 :

$$\tau_1 = \frac{Q}{4\pi T} \quad \text{and} \quad \tau_2 = \frac{Q}{4\pi T} \ln\left(\frac{2.25T}{r_e^2 S}\right) + s_w(Q) \quad (21)$$

Note that $\tau_1 = \Omega$ since $\Omega = \frac{ds_{total}}{dt} t = \frac{ds_{aq}}{dt} t = \frac{Q}{4\pi T t} t = \frac{Q}{4\pi T} = \tau_1$.

Sequentially, the temporal derivative of Equation 20d is:

$$\frac{ds_{total}}{dt} = \frac{\tau_1}{t} \quad (22)$$

which can be ultimately plugged into Equation 18 to quickly estimate T . Note that the derivative solution is not biased by well-loss effects as the s_w and r_e parameters were removed.

3.2 Estimating Jacob’s Well-Loss Coefficient ($n = 2$)

It should be reiterated that s_{total} is the total drawdown due to contributions of head loss from both the aquifer and the well. The exact effects of well loss in the field are not clearly discernable, so there has been much debate on whether Jacob’s or Rorabaugh’s equation uses the more appropriate s_w definition. Jacob’s is supposedly more physically-based but many datasets do not seem to fit his expression, so Rorabaugh presented a model that has an additional

parameter to better fit observed results. Thus, two methods for estimating well-loss parameters are presented. The first method uses Jacob's assumption that $n = 2$. The second employs Rorabaugh's assumption that n is an unknown parameter to be estimated, which will be resolved in the next subsection.

To start, Equation 3, Jacob's expression for s_{total} at a pumping well, has only two unknown parameters— $B(t^*)$ and C_J . Thus, a simple system of two equations can be developed to solve for the two unknowns. The two total drawdown equations are described via two independent constant-rate tests ran at different pumping rates (Q_1 and Q_2). Each constant-rate test is considered “independent” as the water-level in the well returns to its original, static water-level between the two pumping rates. The drawdown equations are then evaluated at the same time after each aquifer test began, t^* , to make the equations directly comparable for subsequent analysis. This can be summarized below while noting $Q_1 < Q_2$:

$$s_{total}(Q_1, t^*) = B(t^*)Q_1 + C_J Q_1^2 \quad (23a)$$

$$s_{total}(Q_2, t^*) = B(t^*)Q_2 + C_J Q_2^2 \quad (23b)$$

The two equations are then normalized by their respective Q s (which produces specific drawdown equations) and subtracted to eliminate one of the unknowns— $B(t^*)$. Here, the specific drawdown equation with the lower pumping rate is subtracted from that of the greater pumping rate:

$$\frac{s_{total}(Q_2, t^*)}{Q_2} - \frac{s_{total}(Q_1, t^*)}{Q_1} = C_J Q_2 - C_J Q_1 \quad (24)$$

To simplify notation:

$$\Delta \left(\frac{s_{total}}{Q_i} \right) \Big|_{2-1} = \frac{s_{total}(Q_2, t^*)}{Q_2} - \frac{s_{total}(Q_1, t^*)}{Q_1} \quad (25)$$

Rearranging Equation 24 ultimately resolves the final solution for Jacob's well-loss coefficient at a pumping well assuming Jacob's assumption that $n = 2$:

$$C_J = \frac{\Delta \left(\frac{s_{total}}{Q_i} \right) \Big|_{2-1}}{Q_2 - Q_1} \quad (26)$$

Equation 26 should be plotted vs t to arrive at the converging value. It is important to realize that the individual specific drawdowns vary in time, but their differences should quickly become constant to result in a single C_J for the analyzed pumping well. Lastly, C_J can be used to complete Jacob's $s_w(Q) = C_J Q^2$ model (Equation 6) to estimate the well loss in a pumping well.

Figure 6 is a conceptual plot of specific drawdowns vs $\log t^*$. The offset between the lines is indicative of well loss. For example, if there was no well loss (i.e., $C_J = 0$) during pumping,

$\frac{s_{total}(Q_1, t^*)}{Q_1} = B(t^*) = \frac{s_{total}(Q_2, t^*)}{Q_2}$, therefore the lines would lay on top of each other. Then, if well

loss increases, so would the offset between the two lines.

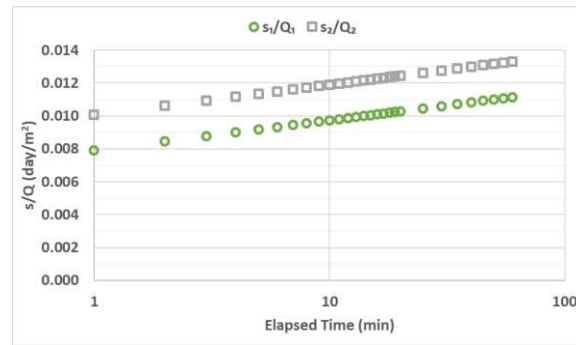


Figure 6 – Conceptual plot of specific drawdown vs $\log t^*$

Since C_J is now resolved, an expression for the other unknown in the system of equations can be developed. Plugging Equation 26 into either drawdown expression (Equation 23a/b) produces:

$$B(t^*) = \frac{s_{total}(Q_1, t^*) Q_2^2 - s_{total}(Q_2, t^*) Q_1^2}{Q_1 Q_2^2 - Q_2 Q_1^2} \quad (27)$$

$B(t^*)$, though, is not needed for the subsequent methods or analyses.

3.3 Estimating Rorabaugh's Well-Loss Coefficient and Well-Loss Exponent ($2 \leq n \leq 2.8$)

This method starts with Equation 4, Rorabaugh's expression for s_{total} at a pumping well. The additional unknown parameter, n , makes the problem more complex. To resolve C_R one must first solve for n , but to solve for n one must first make a critical assumption: the first, lowest pumping rate, Q_1 , is low enough to have $C_R Q_1^n$ be essentially zero. In other words, Q_1 must be small so basically no non-linear losses occur during pumping ($s_w \approx 0$). This assumption relies on the relative magnitude of the two head-loss coefficients in s_{total} . Because C_R tends to be significantly less than $B(t^*)$, as Q gets smaller the s_w term will vanish much more quickly than the s_{aq} term. A practical constant-rate to minimize well loss is assumed to be 10% of the pump's capacity (Hemenway 2018).

Similar to Equation 23, the observed drawdown is evaluated during two independent constant-rate tests, but now Q_1 is assumed to be sufficiently low to neglect the non-linear well-loss term:

$$s_{total}(Q_1, t^*) = B(t^*)Q_1 \quad (28a)$$

$$s_{total}(Q_2, t^*) = B(t^*)Q_2 + C_R Q_2^n \quad (28b)$$

The two equations are normalized by their respective Q and subtracted to eliminate $B(t^*)$:

$$\Delta \left(\frac{s_{total}}{Q_i} \right) \Big|_{2-1} = C_R Q_2^{n-1} \quad (29)$$

Equation 29 is then rearranged for an expression for the C_R between the first and second independent constant-rate tests:

$$C_R = \frac{\Delta \left(\frac{s_{total}}{Q_i} \right) \Big|_{2-1}}{Q_2^{n-1}} \quad (30)$$

Multiplying the right-hand side by $\frac{Q_2}{Q_2}$, Equation 30 is equivalent to:

$$C_R = \frac{Q_2 \left[\Delta \left(\frac{s_{total}}{Q_i} \right) \Big|_{2-1} \right]}{Q_2^n} \quad (31)$$

Since Rorabaugh's expression has three unknowns, a third equation must be introduced to resolve n . The third equation is another s_{total} expression during an even larger pumping rate, Q_3 :

$$s_{total}(Q_3, t^*) = B(t^*)Q_3 + C_R Q_3^n \quad (32)$$

Assuming C_R is a constant between all specific drawdown curves, Equation 31 can be substituted into Equation 32 with the simplifying notation:

$$s_{total}(Q_i, t^*) = s_i(t^*) \quad \text{where } i = 1, 2, \text{ or } 3 \quad (33)$$

and, since the first independent constant-rate test is assumed to experience no well loss:

$$\frac{s_1(t^*)}{Q_1} = B(t^*) \quad (34)$$

along with simplifying the numerator on the right-hand side of Equation 31 as:

$$Q_2 \left[\Delta \left(\frac{s_{total}}{Q_i} \right) \Big|_{2-1} \right] = s_2(t^*) - \frac{Q_2}{Q_1} s_1(t^*) \quad (35)$$

results in:

$$s_3(t^*) = \frac{s_1(t^*)}{Q_1} Q_3 + \left[s_2(t^*) - \frac{Q_2}{Q_1} s_1(t^*) \right] \left(\frac{Q_3}{Q_2} \right)^n \quad (36)$$

Finally, rearranging Equation 36 resolves the expression for Rorabaugh's n :

$$n = \frac{\log \left(\frac{s_3(t^*) - \frac{Q_3}{Q_1} s_1(t^*)}{s_2(t^*) - \frac{Q_2}{Q_1} s_1(t^*)} \right)}{\log \left(\frac{Q_3}{Q_2} \right)} \quad (37)$$

Since n is assumed to be a constant parameter, the two transient variables in the first log operation's numerator ($s_1(t^*)$ and $s_3(t^*)$) and denominator ($s_1(t^*)$ and $s_2(t^*)$) should change at the same rate to result in constant differences. An estimate of n via Equation 37 can now be plugged into Equation 30/Equation 31 to estimate C_R . Together, the two well parameters can be used to complete Rorabaugh's $s_w(Q) = C_R Q^n$ model (Equation 7).

An s_w value can also be estimated from Equation 25 $\left[\Delta \left(\frac{s_{total}}{Q_i} \right) \right]_{2-1} = \frac{s_{total}(Q_2, t^*)}{Q_2} - \frac{s_{total}(Q_1, t^*)}{Q_1}$

with Equation 2 [$s_{total} = s_{aq} + s_w$] substituted in while remembering $s_w(Q_1) \approx 0$:

$$\Delta \left(\frac{s_{total}}{Q_i} \right) \Big|_{2-1} = \frac{s_{aq}(Q_2, t^*) + s_w(Q_2)}{Q_2} - \frac{s_{aq}(Q_1, t^*)}{Q_1} \quad (38)$$

Furthermore, substituting in a part of Equation 5 [$s_{aq}(Q, t) = B(t)Q$] produces:

$$\Delta \left(\frac{s_{total}}{Q_i} \right) \Big|_{2-1} = \frac{B(t^*)Q_2}{Q_2} + \frac{s_w(Q_2)}{Q_2} - \frac{B(t^*)Q_1}{Q_1} \quad (39)$$

or:

$$\Delta \left(\frac{s_{total}}{Q_i} \right) \Big|_{2-1} = \frac{s_w(Q_2)}{Q_2} \quad (40)$$

Rearranging Equation 40 creates a direct expression for the absolute excess drawdown during the second independent constant-rate test (if Q_1 is low enough to assume $s_w(Q_1) \approx 0$):

$$s_w(Q_2) = Q_2 \left[\Delta \left(\frac{s_{total}}{Q_i} \right) \Big|_{2-1} \right] \quad (41)$$

Well loss calculations between the first and third independent constant-rate tests will have an identical derivation. Therefore, Equation 41 can be generalized (and simplified) to an expression for the excess drawdown during an independent constant-rate test with a pumping rate of Q_i :

$$s_w(Q_i) = s_i(t^*) - \frac{Q_i}{Q_1} s_1(t^*) \quad (42)$$

As $s_w(Q_1) \approx 0$, the second term of Equation 42 is synonymous to $B(t^*)Q_i$ but has the advantage of not needing the well's r_e or aquifer's S to be resolved.

3.3.1 Optional n Correction

The simplifying assumption that $s_w \approx 0$ for Rorabaugh's $s_{total}(Q_1, t^*)$ —Equation 28a—instills some error into well parameter estimates. It is not obvious, but n will be overpredicted for all t via Equation 37. The error should not be too great, especially as Q_1 becomes smaller. Still, a polynomial regression model was made to correct n back to some synthetic data's $n_{predefined}$. The synthetic drawdown data (of Rorabaugh's s_{total} expression for three independent constant-rate tests) is introduced in further detail in Section 4.1.1.

Equation 37's modeled n (n_{model}) is a function of the aquifer tests' pumping rates. So, to create the regression equation, the synthetic data was updated by iteratively changing two independent variables—(1) $n_{predefined}$ and (2) $Q_{2\%}$, Q_2 's percentage of Q_3 —for the one dependent variable $n_{corrected}$. Q_1 was taken to be 10% of Q_3 due to practical constraints of the pump's motor and

drive, but $Q_{2\%}$ was allowed to vary in case a project needs to test a particular pumping rate. Changing the synthetic data's predefined C_R , T , and S had no effect on well parameter estimates. Furthermore, the coefficients for $Q_{2\%}$ were found to be statistically insignificant at the 5% level, so they were removed from the final solution for $n_{corrected}$:

$$n_{corrected} = -0.9304 + 1.576 n_{model} - 0.09141 n_{model}^2 \quad (43)$$

The polynomial regression analysis is not consistent with the rest of the proposed methods' theme of being analytical, but it is still simple as there is just one independent variable— n_{model} . The analysis was completed via MATLAB® 2018b and is presented below in Figure 7.

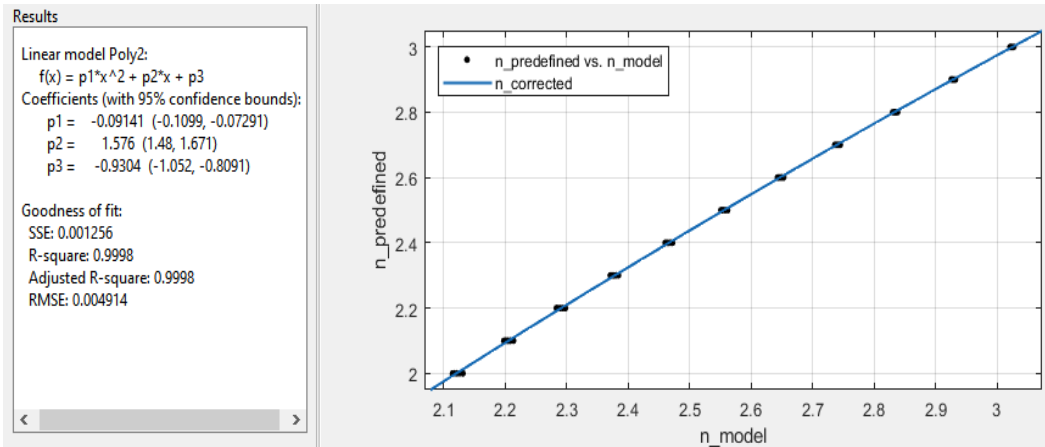


Figure 7 – Polynomial regression model for $n_{corrected}$ given the overpredicted n_{model}

Notice the expression is dimensionless and has a R^2 of 0.9998. Once $n_{corrected}$ is calculated, the final, corrected solution for Rorabaugh's well-loss coefficient at a pumping well is:

$$C_{R_corrected} = \frac{\frac{s_3(t^*)}{Q_3} - \frac{s_2(t^*)}{Q_2}}{Q_3^{n_{corrected}-1} - Q_2^{n_{corrected}-1}} \quad (44)$$

Also note the value of $B(t^*)$ or any of the aquifer parameters were not required to resolve the final $n_{corrected}$ or $C_{R_corrected}$ estimates. Nonetheless, $B(t^*)$ can be estimated via Equation 34.

The author supports a modified set of aquifer tests—either (A) two independent constant-rate tests with different pumping rates or (B) three independent constant-rate tests with different pumping rates where one is sufficiently low for $s_w(Q_1) \approx 0$. Each independent constant-rate test should last at least 60 minutes or, if real-time data are available, until drawdown measurements appear to stabilize. The A set will produce data that can be used to estimate Jacob’s well-loss coefficient. The B set will produce data so Rorabaugh’s well-loss coefficient and well-loss exponent can be estimated. Depending on the set that was employed, a pumping well’s well loss can be easily calculated via Jacob’s or Rorabaugh’s s_w model. Through either method, the subjectivity of standard step-drawdown test analyses can be avoided. Proposing a new type of aquifer test may not seem the most appealing to practice, but an independent constant-rate test is not that atypical.

3.4 Economic and Environmental Impacts of Pumping with Well Loss

Well loss increases the energy required to lift water out of the aquifer, causing a pumping’s economic costs and environmental emissions to concurrently grow. The economic and environmental impacts of well loss are important for highlighting the effects of nonlaminar flows in a pumping well. Correspondingly, discussions of a groundwater well’s life-cycle assessment (LCA) can be taken further, especially for the LCA’s “impact assessment” of a well’s “use phase” (aka its operation).

The additionally required energy to pump the excess drawdown, E_{pump} [$M L^2/T^2$], is given by:

$$E_{pump} = \frac{Q \rho g s_w t_Q}{\varepsilon} \quad (45)$$

where ρ [M/L³] is the density of the groundwater; g [L/T²] is the acceleration due to gravity, which the pump mostly needs to work against to deliver water vertically from the subsurface; t_Q is the duration of pumping; and ε [-] is the combined pump-motor-drive efficiency.

The economic impact (or “cost”) of the extra energy required to pump the well loss, E_{cost} [currency], can be sequentially developed via the cost rate of energy, R_{cost} [currency/(M L²/T²)]:

$$E_{cost} = \frac{Q\rho g s_w t_Q}{\varepsilon} * R_{cost} \quad (46)$$

Similarly, the environmental impact (or “emissions”) of using the extra energy required to pump the excess drawdown, E_{enviro} [M], can be calculated with:

$$E_{enviro} = \frac{Q\rho g s_w}{\varepsilon} * R_{emission} \quad (47)$$

where $R_{emission}$ [M/(M L²/T²)] is the mass emission rate of energy use. The emission rate seems to vary depending on the energy source and efficiencies of producing the electricity for pumping.

Economic analysis can also determine when to rehabilitate a pumping well to ensure efficient well performance. Rehabilitation is suggested when E_{cost} begins to outweigh the cost of rehabilitation (R_{rehab} [currency]). This occurs once the two costs first equate, then exacerbates when the E_{cost} increases due to a growing s_w or larger t_Q of interest. The t_Q of interest depends on the planning and management of the groundwater well. Regardless, the equivalent pumping duration in which the cost of rehabilitation can provide (t_Q^* [T]) is:

$$t_Q^* = \frac{\varepsilon * R_{rehab}}{Q\rho g s_w * R_{cost}} \quad (48)$$

The other parameters in Equation 48 are assumed to remain constant; so, if pumping is desired passed the calculated t_Q^* , then rehabilitation is advised. This can be mathematically stated as:

$$t_Q \geq t_Q^* \quad (49)$$

In other words, if a well's t_Q of interest is equal to or greater than the above t_Q^* , then it is time to rehabilitate the pumping well. The t_Q^* equation conceptually makes sense according to the direct and inverse proportionalities of Equation 48. For example, as the R_{rehab} increases, so does t_Q^* . Likewise, as the well's Q increases, t_Q^* decreases.

4. APPLICATION

The application of the presented methods is illustrated in this section using two datasets. The first is synthetic data utilized to validate the proposed solutions' efficacy. The second is field data to serve as an example of the solutions' application to resolve an aquifer's T and a well's C_J and s_w when n is assumed to be 2. Lastly, the associated economic and environmental implications of the pumping well's s_w are evaluated.

4.1 Available Data

4.1.1 Synthetic Data

This study analyzed synthetic data to determine the accuracy of the proposed solutions. Synthetic drawdown data was produced via the principal equations of Theis (Equation 5) and Rorabaugh (Equation 4) with pumping, aquifer, and well parameters representative of a high-production groundwater well. The predefined aquifer parameters and pumping rates are listed in Table 1. It is assumed a 24-inch diameter well is completed in a confined, unconsolidated fine-sand aquifer that is 100 meters thick.

Table 1 – Synthetic Data Aquifer Parameters and Pumping Rates

Parameter	Value	Units
r_e	0.3	m
S	0.001	--
T	100	m ² /day
Q_1	50	m ³ /day
Q_2	520	m ³ /day
Q_3	550	m ³ /day

Note, Q_3 (the third, largest pumping rate of the three independent constant-rate aquifer tests) was set so s_{total} would not exceed 8 meters after 60 minutes of pumping. It was found that setting Q_2 closer to Q_3 made n results more accurate, as well—the nonlinearity of Equation 37 makes it difficult to determine the reason. Therefore, Q_2 was assigned to be 95% of Q_3 as the smallest pumping rate difference (ΔQ) is assumed to be $\sim 5\%$ by which a variable frequency drive submersible pump can be changed. Lastly, in order to make the $s_w(Q_1) \approx 0$ assumption valid, Q_1 was set to 10% of Q_3 as this is probably the lowest pumping rate ($Q_{min} = 10\% * Q_{max}$) that the same pump can reliably maintain.

As for the predefined well parameters, $C_J = 1.5 \times 10^{-6} \frac{day^2}{m^5}$ when $n = 2$ to depict a severely clogged well (Walton 1962). Appendix I provides a table of Walton’s (1962) criteria of well performance to serve as an easy reference for subsequent analyses. The predefined C_R and n values are provided within Table 2 in Section 4.3.1 when n is allowed to vary.

4.1.2 Field Data

A 72-hour constant-rate aquifer test and a step-drawdown test dataset was collected from a newly installed groundwater production well in the Denver Basin in Colorado by Hemenway Groundwater Engineering. The “field well” taps into the Arapahoe aquifer which is a confined sandstone aquifer with interbedding soft shale layers. The field well was completed with a 12-inch diameter stainless steel v-slot wire wrap screens placed adjacent to the sandstone beds. The field well was drilled via reverse circulation mud rotary then rigorously developed to maintain the hydraulic connection to the aquifer. The field well is equipped with a variable frequency drive submersible pump to ensure constant flowrates during pumping (Hemenway 2018). Figure 2 is a conceptual model for the pumping well and surrounding geology.

The utilized field data consists of high frequency (every two minutes) water level measurements captured via a pressure transducer. Only some of the data was needed for the proposed methods—the first step of a traditional step-drawdown test that lasted 60 minutes (with $Q_1 = 2180 \frac{m^3}{day}$) and the first 60 minutes of the constant-rate test (with $Q_2 = 3820 \frac{m^3}{day}$). The water-level in the well returned to its original, static position between the two tests, so they can be considered “independent constant-rate tests”. The duration of these independent constant-rate tests was short enough that the effects of water-level trends and barometric pressure changes on drawdown data were neglected (Istok and Dawson 1991). All data was smoothed via a running weighted mean, aka hanning. A logarithmic equation was then fit to the smoothed data to create an even smoother drawdown curve. This was completed in Microsoft® Excel via the scatter plot’s trendline function as illustrated in Figure 8. Note the very early data ($t < 10min$) was not included as it showed irregular behavior—probably due to well storage.

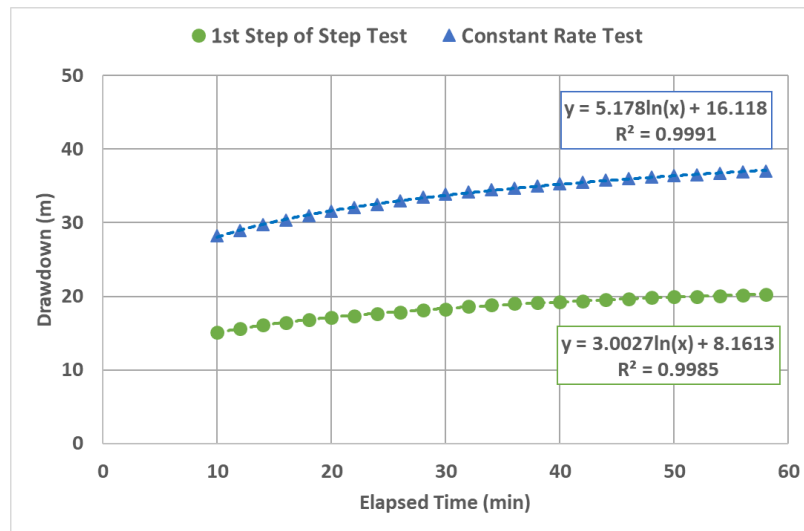


Figure 8 – Step-drawdown and constant-rate drawdown curves with their best-fit equations from a groundwater production well in Castle Rock, CO; $Q_{step_1} = 2180 m^3/day$, $Q_{constant_2} = 3820 m^3/day$

4.2 Aquifer Transmissivity Estimates

4.2.1 Via Derivative Analysis

In order to estimate the synthetic data's T via derivative analysis (Equation 18), the numerical derivatives of total drawdown were calculated via a central finite-difference scheme:

$$\dot{s}(Q, t) \approx \frac{\Delta s_i(Q_i, t)}{\Delta t} = \frac{s_i(Q_i, t_{i+1}) - s_i(Q_i, t_{i-1})}{t_{i+1} - t_{i-1}} \quad (50)$$

where t_{i+1} [T] and t_{i-1} [T] are the t after and before the evaluated t , respectively. This simple scheme is sufficient here as synthetic drawdown data was produced every minute from $t = 1 \text{ min}$ to $t = 20 \text{ min}$ then every five minutes until $t = 60 \text{ min}$. The high-frequency data should not produce substantial errors in the derivative approximations (Straface 2009).

The synthetic data's predefined transmissivity was returned with the proposed methodology despite the presence of well-loss effects ($s_w = 0.45\text{m} = 10\% s_{aq}$ at $t = 60\text{min}$). Equation 18 quickly reached the $T_{pre} = 100 \frac{\text{m}^2}{\text{day}}$ solution with $Q_3 = 550 \frac{\text{m}^3}{\text{day}}$ shown in Figure 9.

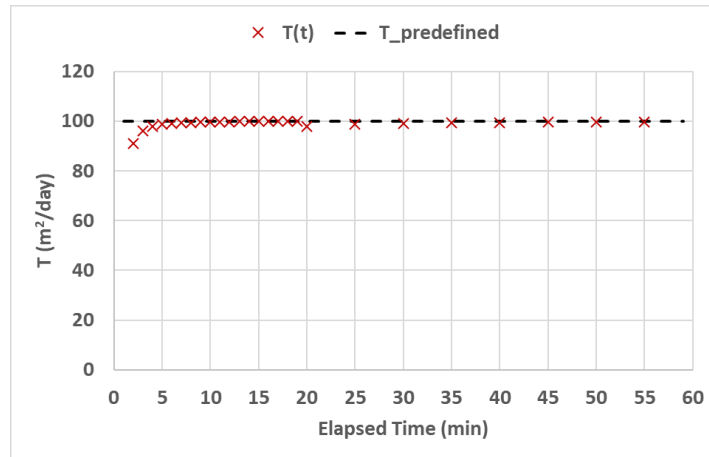


Figure 9 – $T(t)$ estimates of the synthetic data using Equation 18 with $Q_3 = 550 \text{ m}^3/\text{day}$

The synthetic data's $T(t)$ estimates are initially underpredicted for the 1-minute timestep period then the 5-minute timestep period until Ω became constant. The $\frac{\Delta s}{\Delta t}$ needed time to become sufficiently small to equilibrate with the increasing t .

As for the analysis of the field data, the numerical derivatives of drawdown for the first hour of the 72-hour constant-rate test were calculated via the same central finite-difference scheme. This simple scheme is sufficient as the pressure transducer collected high-frequency data (every two minutes), thus there should be no substantial errors in derivative approximations (Straface 2009).

$T(t)$ estimates were plotted during the constant-rate test as illustrated in Figure 10a and appear to be oscillating about an average value. Thus, the 95% confidence interval about the average T was calculated and superimposed on the plot. The field well's narrow 95% confidence interval of T is $60 \pm 2 \frac{m^2}{day}$. The $T(t)$ estimates that lie outside the 95% confidence interval in Figure 10a may be representative of noisy field data and/or violations of Theis' assumptions. For example, the calculated T can change as the cone of depression expands outward from the pumping well and intercepts heterogeneity or boundaries within the aquifer. Lastly, recall the early-time data was not included as it was most likely affected by well storage.

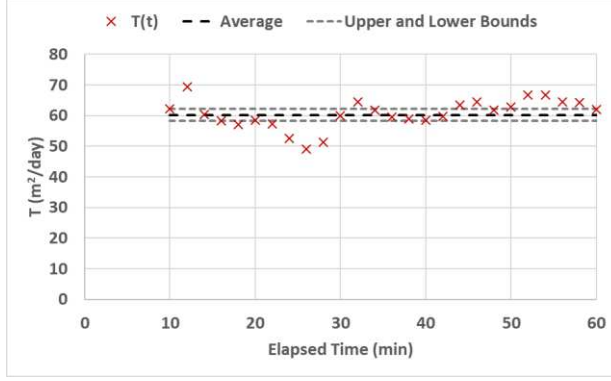


Figure 10a – $T(t)$ estimates, the average, and 95% confidence interval of the field well's constant-rate test using Equation 18

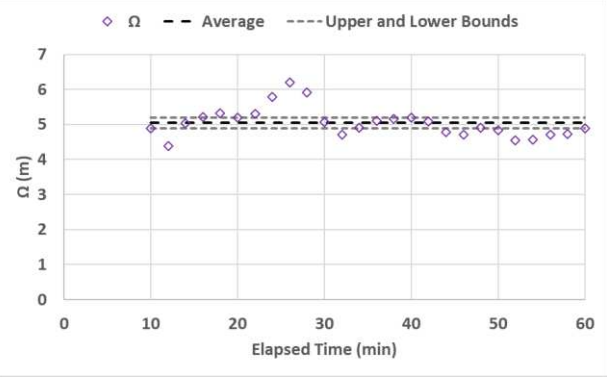


Figure 10b – Ω estimates, the average, and 95% confidence interval of the field well's constant-rate test using Equation 19

The $T(t)$ plot and the Ω vs t plot shown in Figure 10b appear to inversely mirror each other; notice the spikes at $t \approx 25\text{min}$ and the sinusoidal wave starting at $t \approx 35\text{min}$. This makes sense as $T(t)$ is inversely proportional to Ω in Equation 18. It is also important to realize that Ω , a product of two transient parameters, remains essentially constant during the aquifer test. In fact, the field well's narrow 95% confidence interval of Ω is $5 \pm 0.2\text{ m}$.

Errors in the application of these methods should be minimal as most of the time series was used, effectively removing the risk of analyzing an anomalous data point by mistake (Singh 2002). Some standard graphical methods, for instance, recommend selecting only one point in each of the steps of a step-drawdown test, but this increases the likelihood of selecting that erroneous data (Kawecki 1995). Still, it is recommended to report a confidence interval of T to portray the estimate's reliability. Moreover, there may be difficulties in maintaining precise pumping rates in the field if a variable frequency drive submersible pump is not used. Likewise, it might be hard to record accurate water level measurements in the field if a pressure transducer is not used.

4.2.2 Via Fitting a Trendline

The alternative approach to estimate T used Microsoft® Excel to produce a best-fit, total drawdown equation for the field well's 72-hour constant-rate test (as seen in Figure 8):

$$s_{total} = 5.178 m \ln(t) + 16.118 \frac{m}{\ln(min)} \quad (51)$$

whose temporal derivative is:

$$\frac{ds_{total}}{dt} = \frac{ds_{aq}}{dt} = \dot{s}(t) = \frac{5.178 m}{t} \quad (52)$$

which instantly allows:

$$\Omega = \dot{s}(Q, t) t = \tau_1 = 5.178 m \quad (53)$$

to be plugged into Equation 18 to obtain the aquifer's transmissivity about the field well:

$$T = \frac{2.65 \frac{m^3}{day}}{4\pi(5.178 m)} = 59 \frac{m^2}{day} \quad (54)$$

which compares well with the derivative analysis' result of $T = 60 \pm 2 \frac{m^2}{day}$ of the previous section.

It is important to keep most, if not all, the significant figures from the trendline equation throughout calculations since its high precision creates the high R^2 with the smoothed drawdown data. The final estimate of T , however, should be rounded to two or even just one significant figure. Theis' zealous assumptions employed for this analysis do not typically occur in nature—especially that of the aquifer's absolute homogeneous, isotropic nature. Nonetheless, T analyses via this method should be continued with caution if the best-fit equation's R^2 is not particularly high.

4.3 Rorabaugh and Jacob's Well-Loss Coefficient and Well-Loss Exponent

4.3.1 Rorabaugh's Assumption ($2 \leq n \leq 2.8$)

The field well's dataset, unfortunately, could not be analyzed with the proposed, variable- n methods since the Q_1 was not low enough to assume $s_w(Q_1) \approx 0$. Synthetic data was created for verification analysis, anyway. Three independent constant-rate tests at a single well were modeled via Rorabaugh's total drawdown equation (Equation 4) while varying the predefined C_R and n values. Table 1 lists the predefined aquifer parameters and pumping rates. Table 2 lists the predefined well-loss parameters along with the results of the proposed methods—specifically, n estimates via Equation 37 (n_{model}) and $n_{corrected}$ via Equation 43. The results are simply reported because each estimate appeared constant for every t throughout the 60-minute aquifer test. It was also observed that changing the predefined C_R value did not alter the results. This makes sense as Equation 37 and Equation 43 are not functions of C_R . Even further, the results were unaffected when predefined T and r^2S values were varied, which was expected as the $B(t^*)$ terms were also cleared from the final n_{model} and C_R equations.

Table 2 – Predefined Well-Loss Parameters and Solution Results

for all $C_{R_predefined}$		
$n_{predefined}$	n_{model}	$n_{corrected}$
2	2.114	1.993
2.2	2.284	2.192
2.4	2.460	2.394
2.6	2.643	2.596
2.8	2.830	2.798

The synthetic data itself did not make the $C_R Q_1^n \approx 0$ assumption during the first independent constant-rate test. Only the proposed methodology made this assumption in order to resolve an analytical, closed-form solution for n . This discrepancy will not allow the proposed solutions to

exactly reproduce the synthetic data's predefined well parameters. Table 2, in fact, shows that the n_{model} estimates are consistently overpredicted for the range of predefined n ($n_{predefined}$). The worst error between the two is approximately 6% towards the lower end of $n_{predefined}$. Results are more accurate for larger $n_{predefined}$ — n_{model} estimates are even accurate to two significant figures above $n_{predefined} = 2.5$. On the other hand, $n_{corrected}$ values are accurate to at least two significant figures and the largest error is less than 1%. At this point, it is suggested to round $n_{corrected}$ to reproduce the synthetic data's predefined n and C_R .

The proposed $n_{corrected}$ equation was defined via an empirical expression, so it does not go with the rest of the study's theme of analytical solutions. This section's methodology was based on Rorabaugh's total drawdown equation, though, which itself is regarded as an empirical formula. Nonetheless, the proposed solution reproduces accurate and precise results.

It is advised to ensure all units are consistent before applying the proposed methods. The well-loss exponent can be a non-integer now, so conversions after application will not be intuitive. Secondly, it is important to keep in mind the first pumping rate, Q_1 , must be 10% of the third, highest pumping rate, Q_3 , for the proposed methods to be effective. Lastly, remember the empirical expression for $n_{corrected}$ was explicitly found for $Q_{2\%}$ values between 75% and 95%. Although accuracy was only slightly reduced for $Q_{2\%}$ between 50% and 65%.

4.3.2 Standard Step-Drawdown Test Analysis of The Field Well Data

Conventional step-drawdown test analysis of the field well via Jacob's (1947) methodology was conducted to (1) illustrate common sources of error and (2) evaluate the proposed method's estimate of C_J by comparing results. As illustrated in Figure 11, the extrapolated black lines at the end of each step were difficult to ultimately establish as there were multiple possible

combinations of end points that would produce an adequate extrapolation. As the endpoints changed, so did the results via Jacob's methodology. The proposed method, on the other hand, will procure a single C_J estimate that should be obtainable between different personnel performing the calculations. Figure 11 also shows the values for Δs_i and ΔQ_i used in the calculation for the step-drawdown test's C_J [T^2/L^5]:

$$C_J = \frac{\left(\frac{\Delta s_{i-1}}{Q_{i-1}}\right) + \left(\frac{\Delta s_i}{Q_i}\right)}{\Delta Q_{i-1} + \Delta Q_i} \quad (55)$$

Unfortunately, none of the steps reached a stable drawdown before the next pumping rate was initiated. These premature steps would most likely produce an overpredicted estimate of the step-drawdown test's C_J as the steep slopes would make the Δs_i values larger than actual. Therefore, the proposed method's C_J estimate should be less than the step-drawdown test's C_J of $4.6 \times 10^{-7} \frac{\text{day}^2}{\text{m}^5}$, which indicates "mild deterioration" of the well via Walton (1962) in Appendix I. The step-drawdown test's C_J would cause 5.6 meters of excess drawdown when $Q = 3,500 \frac{\text{m}^3}{\text{day}}$.

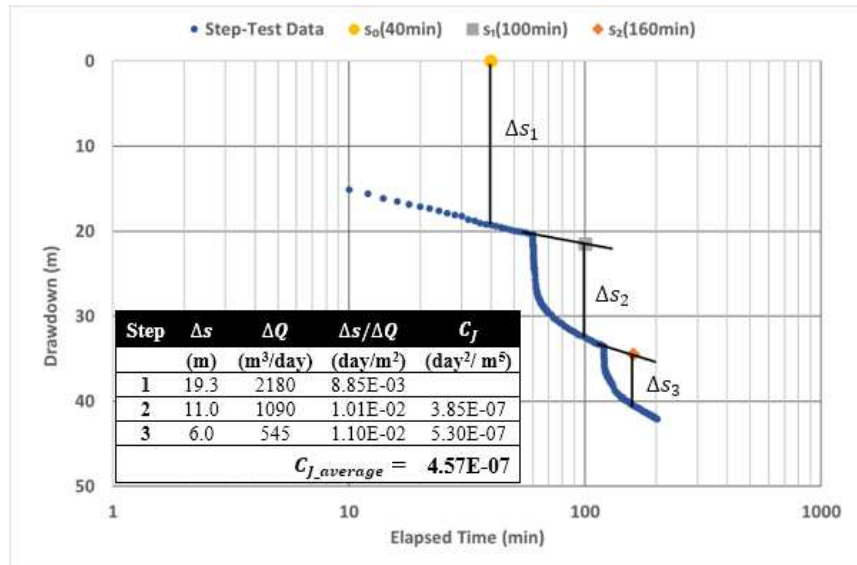


Figure 11 – Jacob's (1947) traditional methodology for the step-drawdown test's C_J

4.3.3 Jacob's Assumption ($n = 2$)

Jacob's well-loss coefficient of the field well was estimated via Equation 26 and the first step (60 minutes long) of a step-drawdown test and the first 60 minutes of a constant-rate test. The water-level returned to its initial, static position between the step and constant-rate tests, so the two can be considered “independent constant-rate tests”. The two tests' specific drawdown curves are plotted in Figure 12a while Equation 26 was used to plot the field well's C_J in Figure 12b.

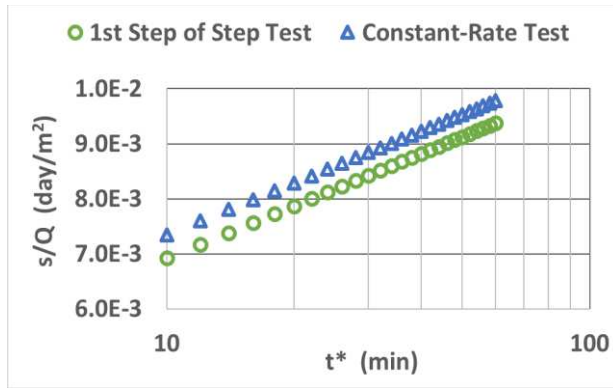


Figure 12a - The field well's s/Q vs t plot

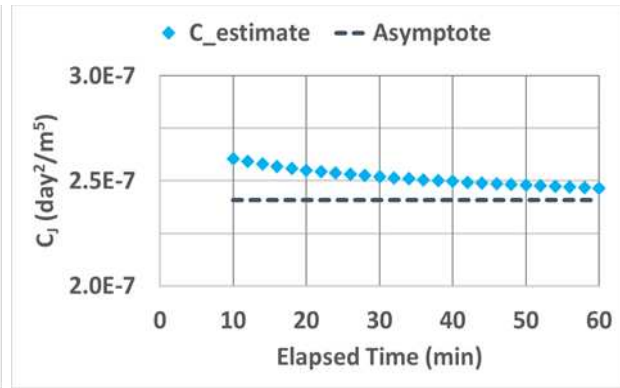


Figure 12b – The field well's C_J vs t plot

Figure 12a was plotted on a semi-log plot to illustrate how s_w affects the specific drawdown lines. It seems well-loss effects are present during the Constant-Rate Test because of the offset between the two $\frac{s_i}{Q_i}$ lines; if there was no well loss, the two lines would overlap. As for Figure 12b, the field well's C_J converges to its lowest value as the differences between the $\frac{s_i}{Q_i}$ lines stabilize. This stabilization occurs when $B(t^*)$ approaches its maximum value in Jacob's total drawdown equation (Equation 23); $B(t^*)$ becomes approximately constant in the field after a sufficiently long period of pumping (Labadie 1975). Therefore, the proposed method's final $C_J = 2.4 \times 10^{-7} \frac{\text{day}^2}{\text{m}^5}$ (which is indeed smaller than Jacob's C of $4.6 \times 10^{-7} \frac{\text{day}^2}{\text{m}^5}$) was anticipated as explained in Section 4.3.2. The presented method's value would be considered appropriate by

Walton's (1962) criteria for a "properly designed and developed well" (Appendix I), because the field well was freshly installed and rigorously developed prior to testing. Lastly, Figure 12b's small standard deviation of C_J was $0.39 \times 10^{-7} \frac{\text{day}^2}{\text{m}^5}$, which strongly supports the accuracy in which C_J was estimated.

As opposed to derivative drawdown data, the absolute, measured drawdown data in the pumping well were used in the proposed method's C_J calculations, so subsequent estimates should not be too sensitive to slight measurement errors or large recording time intervals. Still, high-frequency recordings are recommended to find and remove anomalous datapoints before analysis. Also, remember the early-time data was not included as the drawdown measurements were most likely affected by well storage during this period.

There does not seem to be any other significant limitations of the straightforward methodology to estimate Jacob's well-loss coefficient from a single pumping well while assuming $n = 2$. In fact, several authors support the principle equation in which this method is based—Jacob's total drawdown equation. Clark (1977), specifically, propounds that variations from the equation's form can usually be due to doubtful data or when less than four steps are used for step-drawdown test analyses. Furthermore, Kærsgaard (1982) explicitly finds that Jacob's assumption has a physically basis for use in reasonably homogeneous porous media. Bruin and Hudson (1955) conclude that Jacob's equation is more useful for practical engineering application, even though Rorabaugh presented "the more exact method". Gupta (1989) further advocates the $n = 2$ assumption for quicker analyses.

4.3.4 Economic and Environmental Impacts of Pumping with Well Loss

The field well's calculated s_w (via Equation 6 given $n = 2$), associated economic costs and environmental impacts (via Equation 46 and Equation 47, respectively), and equivalent pumping duration (via Equation 48) given the pumping rate of the 72-hour constant-rate test are provided below in Table 3.

Table 3 – Inputs and Results of Economic and Environmental Impact Analyses of the Field Well's Subsequent s_w (given $n = 2$)

Input	Value	Units	Result	Value	Units
Q	3816	m ³ /day	s_w	0.57	m
ρ	1000	kg/m ³	E_{pump}	3,100	kWh
t_Q	1	year	E_{cost}	310	U.S. dollars
ε	50%	-	E_{enviro}	1,500	kg CO ₂
R_{cost}	10	U.S. cents/kWh	t_Q^*	160	years
$R_{emission}$	0.5	kg CO ₂ /kWh			
R_{rehab}	50,000	U.S. dollars			

Several of the input values are good assumptions for practice but should be adapted to better describe a groundwater well's particular system. For example, ρ is a standard freshwater value, but should be changed to better account for the groundwater's actual temperature and various constituent concentrations. Furthermore, the motor and pump's individual ε are both assumed to be 70%, which results in the 50% combined efficiency; however, more efficient equipment will increase the ε input value. The energy's R_{cost} is the average electricity retail price of the United States in 2017 (U.S. EIA). The environment's $R_{emission}$ is the average carbon dioxide (CO₂) emission factor for the United States' electricity generation in 2017 (U.S. EIA). Electricity prices and emission rates will change, though, depending on the particular energy market, energy source(s) and efficiency of producing the electricity itself. Lastly, R_{rehab} will be highly dependent on the particular well's design, site settings, and condition.

The calculated s_w for the 72-hour constant-rate test is low for a newly installed and rigorously developed well— s_w accounts for approximately just 1.5% of s_{total} after 60 minutes of pumping. The associated economic and environmental impacts of pumping the field well's additional TDH caused by s_w for a year suggest the pumping well is performing efficiently with relatively low impacts. For example, the 1,500 kg of CO₂ emissions is equivalent to just 0.2 U.S. homes' energy use for one year (U.S. EPA). Lastly, the calculated 310 dollars/year to pump the field well's minimal well loss suggests rehabilitation is not necessary given the current conditions and a practical design period of a well system, which is typically less than the estimated $t_Q^* = 160$ years. As a result, Equation 49 [$t_Q \geq t_Q^*$] should not be true. Nevertheless, regular aquifer tests should be conducted every few years to reassess well-loss coefficient estimates due to the perpetual nature of well aging/clogging.

5. CONCLUSION

This section summarizes the motivation behind and the implications of the proposed methods. Suggestions for future work are also presented.

5.1 Summary

Well loss unnecessarily increases the energy required to produce groundwater. Thus, estimating and ensuring well loss is minimized is important for reducing the costs and carbon footprint of pumping. Standard methods exist to quantify the aquifer- and well-loss contributions of drawdown, but these methods have limitations. A few examples include step-test analyses, which are highly subjective, and computer-based techniques, which are rather involved.

Straightforward, less subjective methods were developed to overcome the limitations of standard methods. Specifically, derivative analysis and trendline functions were used to accurately and objectively estimate the aquifer's transmissivity about a single pumping well. The field well's transmissivity had a narrow confidence interval, ensuring the accuracy of the methodologies. Furthermore, groundwater professionals and students analyzing the same drawdown data should obtain the same transmissivity estimates with the proposed methods. Other methodologies were proposed to simply estimate Jacob's well-loss coefficient or Rorabaugh's well-loss coefficient and well-loss exponent via independent constant-rate tests at a single pumping well. More precisely, a simple system of equations, an elementary assumption, and an optional regression model were employed to estimate the well-loss parameter(s). For example, Jacob's well-loss coefficient of the field well embodied a low standard deviation, illustrating the reliability of the method.

The well-loss parameter(s) were, then, used to estimate the excess drawdown due to nonlaminar flows at the design pumping rate. Finally, the equivalent economic costs and environmental emissions of pumping the well loss were resolved. Hence, the proposed methods facilitate the use of well loss as a well performance metric. When well loss and its impacts become too large, further development of a new well/rehabilitation of a preexisting well is advised. The pumping well's development/rehabilitation will reduce well loss, energy requirements, and the associated economic and environmental costs of groundwater production.

5.2 Recommendations for Future Work

This section suggests several potential avenues of future efforts to increase the efficacy of the proposed methods. Applying the solutions to other groundwater wells in a variety of different hydrogeologic conditions can provide further highlights of the methods' advantages. More applications of the proposed methods can also offer insight to other limitations the author is unaware of at this time.

Simply applying the solutions to more groundwater wells completed in a confined aquifer can refine the application of the proposed methods. This argument is taken from a purely statistical standpoint as the methods were applied to only one field dataset. The field well's aquifer test data was very clean. Other wells of similar design in similar aquifers will undoubtedly not provide such ideal data, which can considerably affect the solutions' accuracy. Thankfully, Hemenway Groundwater Engineering is planning to run three independent constant-rate tests at a groundwater production well in Highlands Ranch, Colorado in Summer 2019. The proposed methods should be applied to the upcoming data in an attempt to find the prevalent sources of error and potentially crucial limitations of the methods' application.

Another good step would be to start analyzing unconfined aquifer test data. Theoretically, Theis' equation can be used to describe an unconfined aquifer's drawdown response to pumping once delayed drainage has passed and if the change in the saturated thickness remains small (Theis 1935). It is therefore advised to apply the proposed methods to aquifer test data from a well in an unconfined aquifer to validate the method's diverse applicability.

Lastly, automating the proposed methods seems crucial for the modern world's trajectory towards big-data and energy efficiency (Ronayne et al. 2017; Karimi Askarani et al. 2018). Developing a program with a user-friendly graphical user interface should make estimates of aquifer and well parameters much quicker and more readily available. Analyses of economic and environmental could be standardized, too, for more effective groundwater well installation and rehabilitation efforts. It is therefore recommended to develop a computer-based program for the automated analyses of aquifer test data via the proposed methods.

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APPENDIX I

Table 4 – Walton's (1962) Well Performance Criteria Based on Well-Loss Coefficient Estimates

C_J		Well Performance
sec^2/ft^5	day^2/m^5	
< 5	$< 2.5 \times 10^{-7}$	Properly developed and designed well
5 – 10	$2.5 \times 10^{-7} - 5.1 \times 10^{-7}$	Mild deterioration
10 – 40	$5.1 \times 10^{-7} - 2.0 \times 10^{-6}$	Severe clogging
> 40	$> 2.0 \times 10^{-6}$	Difficult/Impossible rehabilitation

LIST OF ABBREVIATIONS

Abbreviation	Dimensions*	Description
$B(t)$	T/L^2	aquifer-loss coefficient
CO_2	-	carbon dioxide
C_J	T^2/L^5	Jacob's well-loss coefficient
C_R	T^n/L^{3n-1}	Rorabaugh's well-loss coefficient
$C_{R_corrected}$	T^n/L^{3n-1}	refined C_R estimate via Equation 44
$\frac{ds_{total}}{dt}$ or $\dot{s}(Q, t)$	L/T	derivative drawdown
DTW_{static}	L	depth from the well head to the static groundwater level prior to pumping
E_{cost}	currency	economic impact (or "cost") of supplying the extra required energy to pump the excess drawdown
E_{enviro}	M	the environmental impact (or "emissions") of using the extra required energy to pump the excess drawdown
E_{pump}	$M L^2/T^2$	extra required energy to pump the excess drawdown
$E_w(Q, t)$	-	well efficiency
g	L/T^2	acceleration due to gravity
h_f	L	friction losses contributable to conveyance
h_p	L	pressure head required for sufficient discharge at the point of use
h_{use}	L	the elevation change from the well head to the point of use
i	-	index of list [1, 2, or 3]
m	-	variable of integration
n	-	Rorabaugh's well-loss exponent
$n_{corrected}$	-	refined n estimate via Equation 43
n_{model}	-	modeled n via Equation 37
$n_{predefined}$	-	synthetic data's predefined n value
$Q_{2\%}$	-	Q_2 's percentage of Q_3
Q	L^3/T	pumping rate
Q_i	L^3/T	pumping rate during the i th aquifer test
r	L	radial distance from the pumping well to a point of interest
R_{cost}	currency/	cost rate of energy

	(M L ² /T ²)	
r_e	L	effective well radius
$R_{emission}$	M/(M L ² /T ²)	mass emission rate of energy use
R_{rehab}	currency	cost of rehabilitation
S	-	aquifer storage coefficient
s_i	L	total drawdown during the i th aquifer test
$\frac{s_i}{Q_i}$	T/L ²	specific drawdown
$s_{aq}(Q, t)$	L	drawdown associated with pumping water from an aquifer
$s_{total}(Q, t)$	L	total drawdown observed within a pumping well
$s_w(Q)$	L	excess drawdown (or “well loss”) associated with nonlaminar flows through a well during pumping
T	L ² /T	aquifer transmissivity
t	T	elapsed times since pumping began
t^*	T	common, evaluated time after each aquifer test began
t_{i+1}	T	t after the evaluated t
t_{i-1}	T	t before the evaluated t
t_o	T	t at zero-drawdown
t_Q	T	duration of pumping
t_Q^*	T	equivalent pumping duration in which the cost of rehabilitation can provide
TDH	L	total dynamic head
u	-	u
$W(u)$	-	well function
Δ	-	delta
$\frac{\Delta s}{\Delta t}$	L/T	approximated derivative drawdown
ε	-	combined pump-motor-drive efficiency
ρ	M/L ³	density of groundwater
τ_1	L	best-fit equation parameter 1
τ_2	L/ln(T)	best-fit equation parameter 2
Ω	L	omega

* [T = Time; L=Length; M=Mass]